TEACHING PROBLEM SOLVING IN CHEMISTRY - AN EXAMPLE TO STIMULATE DISCUSSION

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As a vehicle to start a general discussion on problem solving in chemistry, this paper presents a course that I taught for six years. This course evolved over the years as I learned more about learning. In the process, I incorporated much of the course material into my teaching in my other classes. As a consequence, my problem solving class has now evolved into a preparatory class for students taking entrance exams to professional schools. An outline of the final iteration of the problem solving course and some examples are given in the following pages.

Introduction

Student Remarks and Questions:

> I got an "A" in high school chemistry. Why am I failing your course?
> I did all the homework problems. Why can't I solve the problems on the exams?
> I understand the classroom example problems, so why are the exam questions so tricky?

Instructor's Thought:

> I don't understand why they don't understand!!!!!!

My selective memory of my student years and the paranoia of being a newly hired faculty member had me wondering what I was doing wrong. Fortunately, for me, I found the lack of problem solving skills in college students was much more universal. The frustration experienced by my students and me in their problem solving abilities led to my developing a course titled, "Problem Solving in Chemistry." The course was based mostly on the work of others; although, where possible, I would use my personal experiences as either a student or an instructor. I simply assembled the gathered information into an order that made some sense to me and attempted to make the class interactive and fun. In solving problems, the activities varied from singular to group and in-class to out-of-class. The students voluntarily (they received participation credit towards their grade) wrote their solution process to the problems on the board, and the class dissected the solution process with some commentary from me. With each set of problems, alternate methods of solving the problem were explored, and failed solutions were always examined. There was no penalty for not solving correctly a classroom or homework problem, and I made a special effort to not attach a negative stigma to incorrect solutions. However, this did not carry over to exam questions. With student's interests in using Web-based materials, I tried to find examples of "teachy stuff" on the Web. However, as noted by several people in a recent on-line conference (1) keeping track of inactive links is sometimes difficult.

The Course

The first section of material dealt with the mind and the brain. This contained a very brief description of the brain and then focused on the ideas of right brain/left brain thinking (2-5), visual and auditory learning (2), theory of multiple intelligences(6), and healthy body/healthy mind(7). Through a series of on-line and in-class evaluations, the students examined how they learned and where their strengths and weaknesses were in learning. One of the assignments was for each student to document how they learned and prepared for their other classes, what was successful and what was not, and insights that they obtained in the problem solving course that were useful in their other classes. This information was then to be included in their term paper due at the end of the semester.

The term papers uniformly stated that the information in the problem solving course was beneficial to their performance in their other course work. I believe some of them said that because they judged that is what I wanted to hear. However, about half of the students in each class did a reasonable job of documenting how they used the information and how it improved their thinking and problem solving skills. In general, this also matched the objective measures (exams) that I gave in the class. What I personally found most interesting were the student comments dealing with the concept of the healthy body/healthy mind. The course material dealt with the current, easily available, scientific information on the effects to the brain and mind by the amount and quality of sleep and rest, sunshine, diet, mental attitude, physical exercise, sex, nicotine, alcohol, and drugs. With each class, a majority of the students reported making life style changes that they ascribed to giving them better physical, emotional, and occasionally spiritual health and that helped them to improve in their academic pursuits.

The second, and major, section dealt with the concepts actually used in problem solving. Initially the concepts were those presented in a book by Arthur Whimbey and Jack Lochhead, titled "Problem Solving and Comprehension." (8) This text also contained two non-chemistry exams that I used to evaluate the problem solving abilities of my students. The material covered reading for improved comprehension, verbal reasoning, using analogies and relationships, analysis of patterns and trends, and deductive and hypothetical thinking. In listening to students over the years, I have noticed that when the vocabulary is foreign or difficult and the concepts are "heavy", i.e. not simple, or there is a lot of math involved the students tend to skip through or over the material. They then wonder why they don't understand the material and can't solve the problems. The section on reading focuses on reading for comprehension. This involves reading out-loud or sub-vocally to slow the reading process down and focus on each word; taking the information in bite-sized bits that are understandable; and, looking up the definitions of words whose meanings are unknown.

The importance of careful reading and using each word appropriately is demonstrated in the solution of the "Liar Problem" that was taken from an article in Parade Magazine, a newspaper insert, written by Marilyn Vos Savant. (9)

> Alice, Ben, and Charlie are inhabitants of an island where you are either a liar or a truth teller and you either always lie or always tell the truth. One day you meet these three individuals: When you ask Alice whether she is a liar, she answers in the local dialect, which you do not understand. When you ask Ben what Alice said, he replies that, She said she is a liar. You then ask Ben about Charlie and Ben replied that, Charlie is a liar, too. Finally Charlie adds, Alice is a truth teller. Is it possible to determine to which group, liars or truth tellers, that Alice, Ben and Charlie individually belong? If so, list each person in the appropriate group.

The importance of reading leads directly into solving **verbal reasoning** problems. The format for each problem solving section was to start with non-chemistry problems and work into chemistry problems. Two verbal reasoning example problems follow.

Jack is slower than Jill but faster than Phil. Phil is slower than Jack but faster than Dephia. Write the names of the four people in order of decreasing speed.

Boyles Law indicates that the volume of a gas is inversely proportional to the gas pressure, when the temperature and the number of moles is held constant. For a temperature of 25C and the same number of moles in each container, develop a pressure relationship for the following gases. The volume of carbon dioxide is twice that of nitrogen dioxide. The volume of sulfur dioxide is one-half that of oxygen. The ratio of pressures for oxygen and carbon dioxide is the inverse of their molecular weight ratio.

Analogies can be useful teaching and learning tools and through development useful in problem solving. As defined in Webster's New World Dictionary (10):

Analogy -
1. similarity in some respect between things otherwise unlike; partial resemblance.
2. an explaining of something by comparing it point by point with something similar.

I use analogies in class to explain various chemical concepts. The students are then assigned the task of developing two of their own chemical analogies and then two additional analogies that would be useful in non-chemistry classes. Then, where appropriate, I point out through the semester where various problem solving approaches are analogous, and how, with experience, we build up a tool box of ideas/concepts/methods/approaches that can be used to solve a variety of problems.

At this point, I introduce my modification of Polya's four-step method to problem solving. (11) I found a copy of the first edition of his book, "How to Solve It", in a used book store. As I've read more literature on problem solving, I have found numerous references to his work. My approach might be called the 4x4 (four-by-four for those uninitiated in four wheel drive vehicles) problem solving approach.

Step 1. **Learn to question.** When you study and when you are problem solving, ask questions! Do I understand the material? Is this concept clear? Are those statements correct? Are the calculations correct? Is there enough information?

Step 2. **Imitate and practice**. We learn by doing and sometimes by imitating. We follow the procedure of the professor or the example problem in the book. Why did they use that approach? Do I understand all the concepts that were used in solving the problem? Do I understand the mechanics that were used in solving the problem? If I use these concepts and mechanics, can I solve other problems that are similar to the example or can I solve more general problems?

Step 3. **Attitude!** Have an interest, in fact a desire, in solving the problem! Am I interested in solving this problem or in learning to solve problems like this, or do I simply want to get this task done? Do I view solving this problem as a challenge or just another hoop to jump through?

Step 4. **Have a Methodical Plan to Solve the Problem**, such as the, **4-Step Problem Solving Method.** Any problem solving method will contain the following four elements.

A. Understand the problem. This is a holistic process and requires right-brain thinking. Time and energy are wasted when you dont understand the problem.

B. Develop a plan. Understand how the various parts of the problem are connected and how the unknown (the answer to the problem) is linked to the data (information concerning the problem)? This, too, is a holistic process; however, elements of the development may require significant analytical or left-brain thinking.

C. Carry out the plan. This is usually a step-by-step process that is very left-brain oriented.

D. Review and discuss completed solution. A review should be done with each step, i.e. check each step. Be open to changing your mind or your plan. Consider alternatives. Reviewing the problem and the solution allows one a final or overall check. It also allows one to enhance the memory of the solution process for skill building in problem solving. The review will require some combination of right- and left-brain thinking skills.

This 4x4 process is actually presented three times in the class. More detail is given with each subsequent presentation. Example problems are given and the method is worked through with each problem. One example problem, without solution, is used is given below.

What is the molar solubility of ZnS, at 25C?

$$ZnS \quad Zn^{2+} + S^{2-} \qquad\qquad pKsp = 24.7$$

However,

$$S_2^{-} + H_2O \quad HS^{-} + OH^{-} \qquad\qquad pK1 = 7.02$$

$HS^- + H_2O \; H_2S + OH^-$ pK2 = 13.9

The 4x4 process is followed by a discussion on **deductive and hypothetical thinking** for problem solving because we need a way to develop our plan. Again turning to Webster's dictionary for definitions:

Deduction - the act or process of deducing; reasoning from a known principle to an unknown, from the general to the specific, or from a premise to a logical conclusion. (Opposite of induction, reasoning from particular facts or individual cases to a general conclusion.)

Hypothetical - in logic; conditional, assumed, supposed.

Fundamental to solving many mathematical and chemical problems is the ability to understand verbal or written statements concerning the progression of information to solve a problem or to take information and work the problem in reverse to determine the starting point. This statement is used to remind the students of where we have been and let them know where we are going. The solution to such problems may be graphical, mathematical, or verbal. Four "approaches" or plans in problem solving are then presented: forward, reverse, hypothetical (assumptions), and "breaking into parts." These approaches can be used separately and in various combinations. Non-chemical and chemical examples are given below.

Forward Thinking

Susans birthday is two days after Tuesday. What day is Susans birthday?

Tuesday Wednesday Thursday Friday Saturday

1 day after 2 days after

Tuesday Tuesday

Therefore, Susans birthday is on Thursday.

What is the concentration of a solution when 25.75 g of NaOH is dissolved in water and diluted to a final volume of 500.0 mL?

Mass Molar Mass Volume Concentration

(25.75g 39.9971 g/mol) 0.5000 L = ?

The concentration of the solution is 1.288 F NaOH.

Reverse Thinking (This is where I'm often accused of writing "trick" questions.)

Friday is two days after Halloween. Which day is Halloween?

Monday Tuesday Wednesday Thursday Friday Saturday

2 days before 1 day before

Or 1 day after 2 days after

Therefore, Halloween is on Wednesday.

How much NaOH must be added to make 250.0 mL of a 0.1000 F solution?

Mass	Molar Mass	Volume		Concentration
?	39.9971 g/mol	0.2500 L	=	0.1000 F

The amount of NaOH is 0.9999 g.

Hypothetical Thinking (In general students are not very good at solving problems when all the information is not presented to them in the problem. The thought of having to look-up, generate, or assume information to solve a problem, from my experience, is a foreign and difficult concept for the student.)

What is the day after four days before tomorrow? Since tomorrow is not defined, I am going to assume today is Friday.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			Today	Tomorrow	
	4th day before	3rd day before	2nd day before	1st day before	
		(day after)			

Defining (assuming) today as Friday, then the answer is Wednesday.

Calculate the pressure in a 2.00 L container when 25.0 moles of hydrogen gas are held at a constant temperature of 35.0 C. Without additional information, one would assume that the gas behaves ideally and that hydrogen is the only gas in the container. Then,

$$P = (n \times R \times T) / V$$

$$P = (25.0 \text{ mol} \times 0.08206 \text{ L atm/K mol} \times 308.15 \text{ K}) / 2.00 \text{ L}$$

$$P = 316.(08) \text{ atm}$$

However, anytime one makes an assumption to solve a problem, once the problem is solved all assumptions or approximations should be checked. In the gas pressure problem, there is no information provided to check if the container held only hydrogen. However, one can use the **van der Waals** Equation to check on the "ideal" nature of the gas. For hydrogen, the two van der waals constants are "a" = 0.244 Atm � L^2/mol^2 and "b" = 0.0266 L/mol. Then,

$$P_{obs} = ((n\ R\ T) / (V - n\ b)) - a\ (n / V)^2$$

$$P_{obs} = ((25.0 \text{ mol x } 0.08206 \text{ L Atm/K mol x } 308.15 \text{ K}) / (2.00 \text{ L} - 25.0 \text{ mol x } 0.0266 \text{ L/mol}) - 0.244 \text{ Atm } L^2/mol^2 \ (25.0 \text{ mol} / 2.00 \text{ L})^2$$

$$P_{obs} = 435.(41) \text{ atm}$$

In this case the assumption concerning the ideal behavior of the gas was obviously false. With the information given, there is no way to confirm the purity of the gas.

Breaking a Problem into Parts One statement that I've heard many times from students is "I don't even know where to start on this problem." This is particularly true with those horrible "word problems" where the student has to read and sort through the information to find what is pertinent. Another element of developing the plan is to understand the information, and that can be done by breaking the problem into parts (bite sized bits.)

Today is Monday. What is 1 day after 3 days before yesterday?

The problem can be broken into four parts.

[Today is Monday.]What is [1 day after] [3 days before] [yesterday]?

Part	1	4	3	2

Part 1

Th F Sa Su M Tu W

Today

Part 2

Th F Sa Su M Tu W

Yesterday

Part 3

Th F Sa Su M Tu W

3 days before yesterday

Part 4

Th F Sa Su M Tu W

1 day after, 3 days before yesterday

So, the answer is Friday.

What is the concentration of the final aqueous solution, when 25.0 g of pure sodium carbonate is dissolved in a solution with a final volume of 500.0 mL, and 10.0 mL of the first solution is diluted to a final volume of 250.0 mL?

[What is the concentration of the final aqueous solution,]

Part 3

when [25.0 g of pure sodium carbonate is dissolved in a solution with a final volume

of 500.0 mL], and

Part 1 and 2

[10.0 mL of the first solution is diluted to a final volume of 250.0 mL]?

Part 3

Part 1

(25.0 g Na_2CO_3) x (1 mol Na_2CO_3 / 105.9888 g Na_2CO_3) = 0.235(87) mol Na_2CO_3

Part 2

(0.235(87) mol Na_2CO_3) / 0.5000 L soln = 0.471(75) M Na_2CO_3

Part 3

M_c x V_c = M_d x V_d

(0.471(75) M Na_2CO_3) x (10.0 mL) = M_d x (250.0 mL)

M_d = 0.0188(70) M Na_2CO_3

Patterns and Trends

The last area explored in the course is that of patterns and trends. This is another area that seems to be little explored by students. Most of their previous experience has been to produce a single answer to a single question. But, what happens when there are multiple pieces of information. Again turning to the Webster's dictionary for starting points:

Pattern 1. a regular, mainly unvarying way of acting or doing.

2. a predictable or prescribed route, movement, etc.

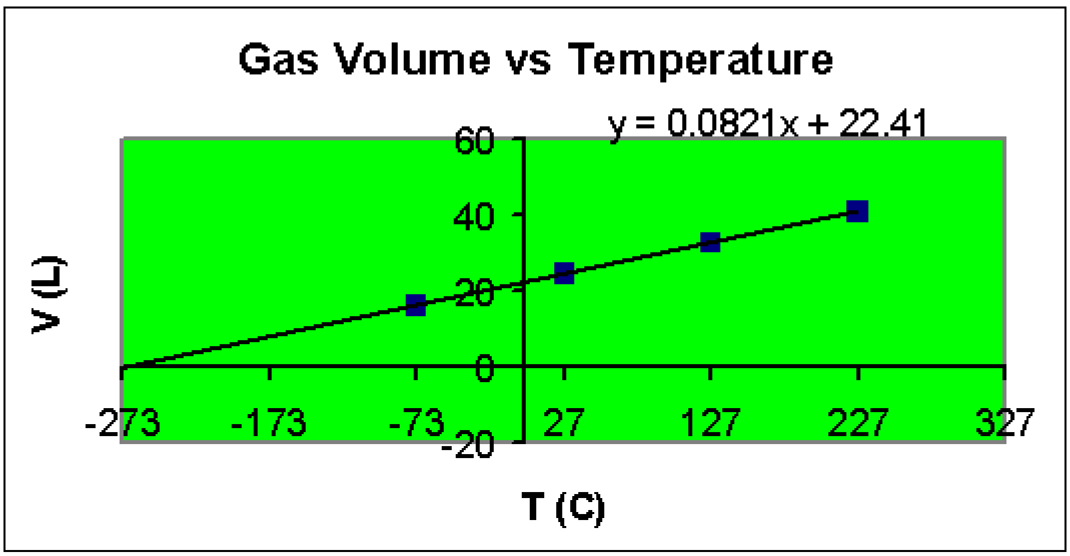
3. grouping or distribution or a diagram showing the distribution.

Trend - the general or prevailing tendency or course, as of events.

In discussing patterns, a series of geometric figures are used where the students must identify the reproducing pattern in the figures. There are typically several different repeating patterns in any one figure depending on how the pattern is defined. This leads into a discussion about long range order, crystal lattices, and unit cells.

In looking at trends, the gas laws are ideal (a bad joke I know), since they have been used several times previously in the class. For example, the French chemist Jacques Alexandre Cesar Charles found that the volume of a gas was proportional to the temperature of the gas. This relationship, known as Charles Law, is an example of a trend. Sometimes, patterns or trends are easier to understand when they can be visualized.

Gas Temp	Gas Volume
(^{0}C)	(L)
226.85	41.03
126.85	32.82
26.85	24.61
-73.15	16.41



A trend line was added to the data points on the graph to extrapolate the data back to zero volume. The equation of the straight line was calculated using a linear regression. Using the equation for the line and setting y = 0, one calculates x = -273.06. The difference between this and the accepted value for absolute zero is due to rounding error in my calculations. Notice also that the slope of the line is that of the Ideal Gas Constant, again with a small amount of rounding error. The y intercept is the volume of 1 mole of an ideal gas at 1 atm and 0C.

Summary

The course started with a non-chemistry test the first day of class and ended with a non-chemistry test during finals week. During the semester two chemistry exams were given. The exams were used to evaluate the improvement in problem solving skills. Typically, the students, who initially scored low on the exams and their self evaluations indicated poor problem solving skills, showed substantial increases in both types of exams. Students with high initial exam scores and self-described good problem solving skills had little if any improvement in exam scores. Unfortunately, as the course became more institutionalized, some students were required to take the class. In general, these students showed little improvement in their problem solving skills as noted by changes in exam scores during the semester.

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