The Chem-Math Project

W. Cary Kilner
University of New Hampshire
wkilner@unh.edu

Students' performance in problem solving, along with their conceptual understanding and subsequent success in general chemistry, can be significantly enhanced by explicit instruction and practice in the specific mathematics skills they will use to solve chemistry problems.

With feedback from teachers I have developed the following 27 *Chem-Math Units.* Note the attention given to the affective domain and other peripheral issues, in conjunction with the acquisition and application of the requisite mathematics skills.

Format for each slide

A) Why this skill is needed, or where it falls in the chemistry syllabus.

B) The pedagogical difficulty, or particular problems this skill constitutes or presents, for students in the chemistry curriculum.

C) Suggested handlings, or the ways I have successfully addressed these problems in my own instruction.

D) Links to available research, and how it informs us regarding this particular pedagogical issue.

Chem-Math Tools

1) Language, Semantics, and Writing to Learn

2) Using the Calculator Appropriately

3) Pattern Recognition

Chem-Math Basics

4) Rearranging Algebraic Expressions

5) Using Scientific Notation (Scaling #1)

6) Metric vs. English Units

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7) Unit Analysis

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Decimal Fractions (Chem-Statistics #2)

Problem-Solving Introduction

13) Authentic Problem-Solving vs. Exercises

14) Mixture-problems (systems of equations) and Weighted Averages (Decoding Word-Problems)

15) Estimation

Algebra Tools

16) Quadratic and Cubic Functions; Geometric Reasoning (Scaling #3)

17) Logarithmic Relationships

18) Inverse-Square Relationships (and Sign Conventions for Potential Energy)

19) Rate-Problems

20) Complex Fractions

Additional Chem-Math Recitation Materials

21) The Use of Proofs and Derivations

22) Formal Statistical Error-Analysis
(Chem-Statistics #3)

23) Problem-Solving Heuristics

24) Developing Metacognitive Strategies

25) Reinforcing Study-Skills

26) Addressing Motivation

27) Using Models

1) Language, Semantics, and Writing to Learn

A) Students must be able to decode the text (as well as word-problems).

They should be able to express their understanding of concepts in their own words.

B) Students' vocabularies are frequently poor. They do not understand the meaning and definitions of physical science nomenclature and of many key technical words used in chemistry.

They don't realize the importance of precise definition.

C) Always define new words as they are used.

Make students use these words in writing.

Always anticipate ambiguity.

Use a collegiate and a technical dictionary during lectures.

Never go past a word you suspect they might misunderstand without clarifying it.

Encourage students to use other texts to cross-reference material.

D) Johnstone and Cassels' classic two studies in the early '80s gives lists of common English words used in science that are frequently misunderstood. Some examples are "burn," "react," "dissolve," "boil," and "substance."

2) ***Using the Calculator Appropriately***

A) Students only know how to use a few operations on their sophisticated graphing calculators.

They are not facile with pencil and paper calculations and have become totally dependent upon calculators for arithmetic operations they should be able to do "in their heads."

B) Students don't utilize their owner's manual.

They don't know how to use the memory function, nor how the calculator stores previous operations.

They don't utilize the very useful "2nd" or "inverse" button.

C) Plan an initial training session with periodic practice sessions to address additional calculator skills as they are needed.

Design exercises that take your students through commonly-used operations.

D) To the degree that students struggle with their calculators they will not have that energy available to solve chemistry problems.

3) ***Pattern Recognition***

A) This is an important link between mathematical problem solving and conceptual understanding; between algorithmic problem solving (“plug ‘n chug”) and conceptual problem solving (problem solving with understanding).

B) Pattern recognition is a vital part of chemistry (e.g. the Periodic Law, formula-writing, identifying reaction types).

Some students have a native intuition for pattern recognition. Others can develop it with proper nurturing.

C) Show students how to discern visual, verbal, and mathematical patterns in various contexts. Provide them with exceptions to various patterns (anomalies) so they can see that exceptions do not necessarily negate emerging patterns. They will not then reject patterns out of hand when they encounter apparent contradictions.

D) There is a large body of research on pattern recognition in cognitive science and cybernetics, and some in mathematics education (Silver), but little in the problem-solving literature.

4) <u>Rearranging Algebraic Expressions</u>

A) This is a foundation chem-math and pre-algebra/algebra I skill.

B) Students can't solve for a divisor or divide by a fraction.

They forget their order of operations, the commutative law, the distributive law, and distributive properties. They don't understand common Greek symbols used in mathematics, science, and engineering.

C) Administer an initial diagnostic quiz of basic algebra skills. Then do lots of drill and practice, remediating as necessary.

Practice rearranging chemistry functions as math equations without numbers, using the proper symbols (rather than "x's").

Derive cross-multiplication with your students to demonstrate how it comes from correct algebra.

When students use the combined gas-law, have them group similar variables together (P_2 /P_1), rather than randomly distributed within the function, in order to see the proportional relationship.

Point out that symbols used for various quantities may be different with other texts and teachers, e.g. Q vs. C_p, q vs ΔH, and the various upper and lower case "K's:" K_{eq} vs. the rate-law "k" vs. the Beer's Law "k."

Give students a handout of the Greek alphabet. Have them practice writing these symbols and help them find them on the keyboard.

D) Many students memorize their algebra without attaining a full conceptual understanding. They don't understand the meanings of "function," "variable," equation," and "mathematical expression." (Kilpatrick, Sfard, Schoenfeld and the NCTM Yearbook '88: *The Ideas of Algebra K-12)*

5) ***Scientific Notation (Scaling #1)***

A) Scientific notation is an essential tool of measurement. Scaling is a *Big-Idea* found in the National Science Education Standards.

B) Students don't understand "orders of magnitude;" they don't understand exponential scaling by tens. Students don't know their Laws of Exponents, or can't transfer them from their abstract mathematics courses to science applications. They insist upon writing out numbers from scientific notation before operating on them. They don't understand "standard-form," and write the calculator "E-notation." They confuse this "E" with base e. They write "$10E^x$" and their answers are off by a factor of 10.

C) Don't allow students to write E-notation; teach them how to translate E-notation into standard form. Instead of a rote rule for moving the decimal, tell students "when you put a number into standard-form, if you make the digits number smaller use a larger power of ten," and "if you make the digits number larger use a smaller power of ten." Thoroughly review the Law of Exponents, especially division with negative exponents. Show the *Powers of Ten* video.

D) Scaling is a component of proportional reasoning (NSES). It is an higher-order cognitive skill (HOCS). The NCTM standards don't include exponential notation.

6) ***Metric vs. English Units***

A) The SI system and metric measure is a rite-of-passage into all of the sciences.

B) Our students have no intuitive feel for metric measure because they have grown up using English units.

C) Don’t enforce “metric-only” rules, and have students compare equivalent units. Do introductory measurement activities of weight (force), temperature, and length, and calculate area and volume.

Have students develop rule-of-thumb approximations for common conversion factors. Give them a table of conversion factors (both SI and non-SI) they will use in chemical calculations.

D) Most students have not manipulated, measured, and compared a sufficient number of physical quantities in their K-12 experience. They don’t develop the intuitive feel for measurement that we expect for upper-level courses. Arons and Epstein have written extensively on activities to assist students in acquiring HOCS. See the IPS program for a well-tested model (Haber-Schaim), and the NCTM Yearbook ’03: *Learning and Teaching Measurement.*

7) ***Unit-Analysis***

A) UA is a very useful algorithm for scientific and engineering work and is vital for understanding derived units. It can direct a computational process. Using multistep UA minimizes the accumulation of rounding errors.

B) Many students learn to use UA mechanically without ever understanding *why* it works. Others never use it correctly; they don't understand *how* it works. Students use slanted lines that gradually degenerate into vertical lines so they lose track of the denominators and thereby miss a central purpose of the process. Mathematics teachers seldom use units and don't teach UA. They teach abstract mathematics as its own subject.

C) Introduce UA early in your course. Use nonsense units as well as authentic exercises. Always require units with measurements. Teach that a "unit" is an algebraic variable and is manipulated as such.

D) UA encourages many students to take a mechanical approach to problem solving. It can negate our best efforts at encouraging conceptual understanding unless we simultaneously reinforce proportional reasoning (Arons, Schoenfeld).

8) <u>Unitary-rates and Derived Units</u>

A) These functions define intensive properties, and also indicate the source of the variable.

B) Students don’t know what “per” means. Their slanted lines degenerate into vertical lines. (Point out that slanted divisor lines are acceptable in word-processing to eliminate justifying the following line of text.)

C) Provide a list of unitary-rates from physics and chemistry: velocity, acceleration, density, pressure, work, and power. Use UA to show how common SI units, such as Pascal, Joule, Watt, are derived from definition. Repeat this derivation whenever students encounter a new unitary-rate. Give them some mass and volume data for samples of a substance and have them graph M vs.V. Point out that the resulting slope represents the density (a unitary-rate) of that substance. Also show that unitary rates can be constants of proportionality, e.g. the Coulomb’s Law constant, R, molar absorbance, etc.

D) Unitary rate is another facet of proportional reasoning. Many students never understand what a “rate” represents, despite the NCTM change from “slope” to “rate” in the new mathematics standards (Arons). See the NCTM Yearbook ’91: *Professional Standards for Teaching Mathematics*.

9) Ratio/Proportional Reasoning (Scaling #2)

A) This skill represents formal reasoning in Piagetian terms. It is the basis for all of quantitative chemistry.

B) Students don't understand "lowest whole- number ratio." Students can't distinguish between direct and inverse variation. Students can't write a mathematical function to represent a physical reality. Given "there are 6 times as many students as teachers," students write "6•S = T." Calculating with proportionality is a very different skill from calculating with UA.

C) Introduce the three gas-laws, presenting them as two quotients and a product that each equal a constant. Then write them as the combined gas-law and show that the interaction of the three variables produces a constant of proportionality. Give lots of examples of proportions from physics as well as from chemistry. Make sure students understand "specific gravity" and "specific heat" as ratios, and can see that they are unitless.

D) Consult the NCTM standards concerning proportional reasoning in the K-12 math continuum, as well as the NCTM Yearbook '02: *Making Sense of Fraction, Ratios, and Proportions,* and the NCTM Yearbook '85: *The Secondary School Mathematics Curriculum.* There is a large body of research on proportional reasoning as a HOCS. Students must construct this understanding, acquired through hands-on exploration. It requires content knowledge as well as PCK from teachers throughout the K-16 pipeline (Bunce, Shulman, Arons, Papert, Karplus, Lawson, Renner).

10) Scientific Graphing

A) Understanding graphs is a cross-disciplinary skill, a vital scientific tool, and an essential component of proportional reasoning.

B) Students enjoy drawing graphs. They draw on autopilot, gleefully going through mechanical actions without understanding. They don't title or spread them out, they don't label their axes and they have trouble scaling them correctly. They use pens and make a mess, and they feel compelled to place "x," "y," and "(0,0)" on their graphs. They don't understand that the slope represents a unitary-rate. They can't identify the dependent and independent variables even though they can define them. They don't know what residuals are when they use their TI-83+ to compute the curve-of-best-fit. They don't understand interpolation and extrapolation.

C) Provide a strict list of graphing rules to follow. Give students sample data to graph by hand. Have them interpolate and extrapolate to obtain data. Have them graph a nonlinear function by hand, such as VP vs.T, using a French curve. Show them how to transform it into a linear function. Teach the basics of linear regression.

D) Note differences in expressing the slope-intercept formula, $y = mx + b$ vs. $y = a + bx$, depending on the text. Students can graph ordered pairs but fail to comprehend their physical meaning. Students confuse the y-intercept with the x-intercept. They don't understand the meaning of "slope." They can't utilize values from a graph to form a correct interpretation of the physical representation (Schoenfeld, Silver, Arons, Epstein).

11) Significant Figures (Chem-Statistics #1)

A) This is a vital concept and *Big-Idea* in the NSES.

B) Students mindlessly report all digits from calculators, and drop significant zeros after a decimal. They consider every measurement and calculation to be exact. They confuse the everyday use of “precision” and of “accuracy” with their technical meaning. Students have had little contact with formal statistics despite the recommendation of the NCTM Standards. Many students continually struggle to understand significant figures.

C) Have students refer to the rules in the textbook whenever they compute, and understanding will come with repeated practice and exposure. Teach the rules for expressing measurements in one sitting and the rules for rounding calculations in another. Tell them to round to the least number of digits used in a problem and it will work most of the time. Give them a worksheet with some ambiguous examples. This will show students why we need the rules and how they work. In performing measurement activities, make sure they are aware of the limits of precision of their measuring instruments. Some good activities are calculating their individual horsepower, the speed of sound, and the period of a pendulum. (These are also good for exploring unitary-rates.) For advanced students, explore the propagation and accumulation of error in sequential calculations.

D) Students taught significant-figures didactically will not understand until given frequent experimental measurement situations in which to make rounding decisions. They will not understand the propagation of error until they see it for themselves. JCE has quite a few references on the teaching of significant figures.

12) *Percent-Composition, Percent-Error, and Decimal Fractions (Chem-Statistics #2)*

A) These concepts are important for overall numeracy, and for chemistry as the study of the composition of matter.

B) Students don't understand what "per-cent" means. They don't know what a "decimal fraction" is; they don't understand mass-fraction, mole-fraction, or volume-fraction, nor how these become percents. They don't understand that "percent composition" means the percent of each constituent in a mixture, or element in a compound. They use a calculator to multiply by 100. They can't calculate the whole when given the part and the percent.

C) Show students that "percent" means "parts-per-hundred," that "%" is a unit that means "pph." Remind them that "per" means a ratio. Percent is the number out of the one hundred parts that constitute the whole. Have them use scientific notation in this manner: x% = decimal-fraction•10^2 %. Define ppm, ppb, and ppt similarly, expressing them in scientific notation. Teach percent-error as the accuracy of a result—a comparison. Use absolute value to show it as the deviation from an expected experimental result, where the original sign tells which way it deviates. Define and distinguish between the validity and reliability of a measurement. Ensure students understand the concept of calibration; have them calibrate thermometers in ice-water. Consult the NCTM standards and reinforce the teaching of statistics. If you introduce standard deviation, use it frequently.

D) Mature students still struggle with ratios. Students do not know and are never instructed as to what the prefix, "per," means (Johnstone, Arons).

13) ***Authentic Problem-Solving vs. Exercises***

A) This is the critical-thinking skill representing formal reasoning sought by all science educators. “A ‘problem’ is what you do when you don’t know what to do” (Grayson Wheatley via George Bodner). “A problem that’s not a problem isn’t a problem” (H. Bent). “Whenever there is a gap between where you are now and where you want to be, and you don’t know how to find a way to cross that gap, you have a problem” (John Hayes).

B) Students learn algorithms but are unable to apply them to novel settings. They cannot see the stages in a multi-step word-problem and bog down easily. They dive in without making a plan. They can’t filter out the “noise” in word problems. They don’t understand “chunking” as information-sequencing. They don’t look for similarities to previous problems they have solved. They don’t use an order-of-magnitude approximation as an answer check. They get frustrated; unwilling to start over and try other stuff out, they give up easily. They don’t review their solutions to see what understanding they acquired from successfully solving a hard problem (Schoenfeld).

C) Collect good problems from a multitude of texts, and organize them to design a gradient of relevant transfer problems that unfolds with the syllabus. Students can gradually develop confidence, facility, and expertise through extended practice. A good example is to calculate the dilution of a concentrated acid to obtain a desired molarity, given the percent acid and density of the concentrate. Continually connect the electronic, molecular, and molar perspectives (Bill Jensen). Teach these classic steps: understand the problem, make a plan, execute the plan, and look back to see what you did.

D) There is lots of published research on transfer-problems, the inability of students to move from exercises to problems, and the useless and haphazard approaches students invent (Schoenfeld, Bunch, Bodner, Nakhleh, Polya). Much discussion is found in inquiry-based instructional materials (Arons, Epstein). It is also found in the knowledge transfer literature (Mestre, et.al.). See the NCTM Yearbook ’80: *Problem Solving in School Mathematics.*

14) Mixture-Problems (Systems of Equations) and Weighted Averages

A) These are common word-problems found throughout the math and science curriculum after Grade 6. They are used to develop proportional reasoning. Some typical examples involve the price and composition of trail-mix, the density and composition of a 2-component alloy, isotopic composition and atomic weights, and found in dilution problems, calorimetry problems, and "marathon" stoichiometry problems.

B) Students don't see the proportional relationships that are contained therein, nor the requisite concept of weighted-averages. They trash the algebra and give up, or accept negative (nonsense) values without evaluation.

C) Collect your own package of mixture problems. Develop skill by using them on a regular basis. Have students work in threes on portable white-boards and demonstrate various student solutions to the whole class. Have them work in pairs using the "think-aloud paired problem-solving" technique (TAPPS). Record audio for later analysis (with student permission). Teach the use of all four NCTM-recommended representations: numerical, graphical, symbolic, and verbal. Remind students that they can use their TI-83 matrix function to solve systems of equations.

D) Mixture problems are used in middle school mathematics but are not effectively followed up in high school. See the NCTM Yearbook '82: *Mathematics for the Middle Grades (5-9).* These problems force students to use proportional-reasoning because they must deal with two ratios embedded in one situation. These are no longer called "simultaneous equations," or "two equations in two unknowns," so students will not recognize them as such.

15) Estimation

A) When solving difficult and multi-step problems it is important to at least do an order-of-magnitude estimation of an answer to ensure the result is at least in the proper “ballpark.”

B) Students often produce ridiculous answers, way out of range or with a sign change, and accept them without inspection. Students think that any answer is better than no answer at all, whether it is meaningful or not. Teachers often just plain give up on this issue out of frustration (one can only nag so much).

C) Teach students how to perform Fermi problems and give them one daily for a while. This will give them practice in estimating orders-of-magnitude. In addition it will be a fun way to provoke thought and induce some critical thinking. Remind them how to add and subtract exponents to estimate a result.

D) This has always been a big problem that is frequently discussed in the problem-solving literature.

16) ***Quadratic and Cubic Functions; Geometric Reasoning (Scaling #3)***

A) This is another bridge between Algebra II and Chemistry and Physics. It constitutes an important understanding throughout the STEM disciplines, and represents another aspect of proportional-reasoning.

B) Students don't understand the "pizza-problem;" they don't understand nonlinear growth. They don't remember the formulas for surface area and volume of spheres and rectilinear solids. They see too few examples of these functions, since they are not grouped together in the syllabus and arise only occasionally. Students don't know where pi comes from and that it's a constant of proportionality.

C) Differentiate these problems from the linear functions with which students are more familiar. Use examples like the satellite dish/size of a telescope problem, and the heating of a large vs. a small building. Compare the area and volume of a cell vs. its radius. Calculate the force crushing the old alcohol can filled with condensing steam, and holding the Magneburg Spheres together after you evacuate them. Students actually welcome a chance to finally use the quadratic formula for something.

D) Lack of opportunities to practice these relationships across disciplines means that students cannot utilize their previous studies in geometry (Polya, Schoenfeld, Arons). See the NCTM publication: *A Call for Change* (1991).

17) Logarithmic Relationships

A) Some examples of logarithmic scales are the Richter scale, the decibel scale, the pH scale, the Beaufort wind velocity scale (in arbitrary scale units), and the magnitude of stars (this base = 2.5, not 10).

B) Students don't transfer understanding from their Algebra II course to these relationships, and perform their calculations mechanically. They don't see the use of log-scales on a graph as a form of scaling. They want everything to be linear.

C) Reteach logarithms as necessary. Show the *Powers of Ten* video again. Ensure students are comfortable using whole-number logarithms before using decimal logarithms. Review the Law of Exponents.

Hand out log-tables and have students graph the log vs. the number on regular graph paper, then graph them again on semi-log paper and explain what is happening. Have them graph a table of natural logs and have them explain why it looks similar to their previous graph. Ask them if graphing the logarithms of any base would look different. Show them how to use a proportional-parts table to obtain more precise log-values. Have them see how this table could be generated through interpolation of their graph of the log table. Give them log-log paper and an appropriate function, then have them graph it and explain why it becomes linear.

D) Science teachers don't generate opportunities to extend students' applications of logarithms into their disciplines. Conversely a lack of understanding of physical science applications means that mathematics teachers can't show how these functions are applied in the sciences.

18) Inverse-Square Relationships (and Sign Conventions for Potential Energy)

A) Some examples are Newton's Law of Universal Gravitation, "candle problems" (light intensity), sound intensity, Coulomb's Law, the period of the pendulum, and Graham's Law of Effusion.

B) While students may understand this function as an abstraction from Algebra II, they can't transfer it to these physical applications. Most texts give a very perfunctory treatment of Coulomb's Law, a very important *Big-Idea* in chemistry. In Universal Gravitation, E_p goes from a value of 0 at reference to a positive number at infinity, whereas in atomic ionization energies, E_p goes from a minimum negative value at reference to 0 at infinity.

C) Clarify these reference conventions for your students, since no one else will likely have done so. Design a pictorial worksheet. Have students graph the above functions, and in the correct quadrant. Review the physics: derive work from f = m•a and W = f•d to show where the units come from. Show how E_p becomes an *inverse* function, while force becomes an *inverse-square* function. Derive Graham's Law, then have your students do the classic demonstration as an experiment for themselves with ammonia and HCl. Compare Coulomb's Law and Universal Gravitation, but give their relative magnitudes by reference to the Four-Forces so that students understand the very different contexts in which these two functions are applicable. Make sure you clarify the sign convention for each.

D) The Chemical Bond Approach (CBA) high school chemistry text (1964) does an excellent job of presenting Coulomb's Law early, and referring back to it frequently in subsequent chapters. There is some study of student difficulties with inverse functions in the mathematics education literature.

19) Rate-Problems

A) Some examples of exponential decay functions are radio-dating half-life problems and drug-dose half-lives, Newton's Law of Cooling, Beer's Law using %T rather than absorbance, rinse problems, and first-order kinetics (and enzyme kinetics from biology).

B) These functions require more than a superficial understanding of the mathematics, so it is hard for students to use an algorithmic approach as a crutch. Students are not asked to derive these laws using experimental observations or tabulated data along with the calculus they are studying, so they don't see the connections. Mathematics teachers fail to use these rich applications of the calculus in their own instruction. Students don't recognize an asymptotic relationship; they want the curve to hit the axis somewhere.

C) This is a great place to integrate mathematics and science. Kinetics is important in all of the science disciplines. Have students perform the classic 100 pennies activity, graphing the results.

D) In the 1990's Texas Instruments and Ohio State University teamed up in the T^3 CasCalc Project to foster the use of technology to enrich mathematics instruction. There is much research in calculus teaching and learning in these materials (Demana and Waits).

20) ***Complex Fractions***

A) Some examples are the Clausius-Clapeyron equation, the van’t Hoff equation and the related Arrhenius equation for activation energy, the Rydberg equation, calculating the reduced mass of a system, and finding the total resistance in parallel circuits and the total capacitance in series circuits.

B) Although students learned complex fractions in Algebra I, they have probably not used them in science. They will need this skill to calculate the energy of excited-state transitions in UV-visible spectroscopy, and in physics when they study electrical circuits and draw free-body diagrams.

C) With all the time intervening you will need to review the mathematics, and perhaps give a pure mathematics worksheet on this skill.

D) See the NCTM Yearbook ’88: *The Ideas of Algebra K-12.*

21) The Use of Proofs and Derivations

A) Proofs and derivations have always been de rigueur in the traditional mathematics curriculum for developing enduring understandings. Doing these in chemistry whenever possible moves students away from mechanical approaches and towards a deeper understanding.

B) When students don't understand what variables represent, or what an overall chemical equation or function represents, they memorize formulas. Then they "plug-'n-chug" on a survival basis despite our best efforts at teaching for understanding. It's hard to memorize a proof or derivation without understanding it. Repeated exposure to such proofs and derivations is necessary, since very little of this approach is found in contemporary mathematics and science coursework.

C) Look for opportunities to present derivations and proofs in your syllabus, particularly in the gas-laws and the KMT units, and within the K_a/K_b weak electrolytes unit.

Present the combined gas-law, then "derive" the three individual laws as special cases by holding a variable constant. Derive the ideal gas-law from the combined gas-law and molar-volume. Solve it for "R," then use units to check the solution. Convert "R" to energy units, and have them see that P•V is an energy term. Derive the gas-density formula from the density formula ($d = MM/V_m$), and have students use it in place of $PV = nRT$ at STP. Derive Graham's Law from the formula for kinetic energy. Derive a formula for molecular weight from U_{rms} (or vice versa). Show Avogadro's proof of diatomic molecules, using HCl, H_2O, and NH_3. Derive K_a for HOAc from K_{eq} for water and K for ionization; derive K_b for NH_3 from K_{eq} for water and K for ionization; derive K_h for NH_4Cl from K_b for NH_3, and K_h for NaOAc from K_a for HOAc. Use pattern recognition since $K_a = K_b = 1.8 \cdot 10^{-5}$ for this weak acid and weak base. Calculate the molarity of water in a dilute solution. Draw four "thermometer scales" and label them. Have students use them to visually derive the formulas for interconverting °C and °F, as well as for Kelvins and degrees Rankine. Then students can derive a new temperature scale based upon the freezing and boiling points of benzene (Silberberg problem #1.96 in the 4th ed).

D) There is still controversy regarding the role of proofs in mathematics instruction. This parallels the Saxon return-to-basics and the phonics vs. whole-language arguments over the issue of how teachers and curriculum developers can best find middle ground. The instructional model of using proofs does not seem to have reached chemistry and physics educators (Sherin).

22) *Formal Statistical Error Analysis (Chem-Statistics #3)*

A) Doing rigorous science requires an understanding of the reliability and validity of data. It is necessary to know the kinds of errors and their relative magnitudes, as well as ways to control error.

B) Middle school science teachers often neglect error analyses of experiments, other than to acknowledge the presence of error. High school science teachers may have students calculate a "percent error" without determining a possible source. Despite much instruction, students do not understand the control of variables (Arons). They associate simultaneity with causation. They think correlation means causation. They don't understand the propagation of error. They don't conceptualize the difference between random and systematic error. Neglecting significant figures, they will express too little or too much error in their calculations.

C) If students have had some rudimentary instruction in statistics, they must still be shown how to apply it to actual experiments. Every quantitative experiment should be accompanied by an attempt to ascribe sources of error and to formulate error bounds. At least one preliminary experiment dealing with measurement, some kind of calibration, and an estimation and calculation of error, should be done early in every science course.

D) Students have had little exposure to useful statistics despite the NCTM efforts to bring it into the mathematics curriculum. *The Journal of College Science Teaching, The Physics Teacher, The Science Teacher,* and *The Journal of Chemical Education* all have articles on analyzing error in experiments. The best references are by Youden, Young, and Baird. See the NCTM Yearbook '81: *Teaching Statistics and Probability.*

23) ***Problem-Solving Heuristics***

A) Poor organizational and representational habits wreak havoc upon students striving to learn new skills in new settings.

B) Students tend to write way too small and can’t see their work clearly. They lay out their work as if writing an essay, instead of giving themselves room to work stuff out.

No one may have ever shown them the iterative process of problem solving. They use pen to write, cross stuff out, make a mess, and then can’t see their work clearly or follow what they have done. This visual disorganization contributes to cognitive disorganization. They do their work in the spaces and margins of anything handed out to them instead of starting their work fresh on blank paper. They don’t methodically rearrange any necessary formulas before plugging in numbers. They just start putting numbers in without units and it all goes to hell.

C) Under constant scrutiny they will develop better work habits. But if not drilled sufficiently, they will revert to their bad habits when left to their own devices.

Tell students to write LARGE! Provide pencils, a pencil sharpener, and don’t let them use pens. Tell them to use pencil so they can erase. Hand out lots of scratch paper so they will use it! Make your worksheets so compact that they can’t begin to do work on them, and write on the top to “use fresh paper for your work.” Nag, nag, nag until students consistently set up their work neatly and methodically. Remember that you are combating years of sloppiness.

D) Most problem-solving studies concentrate upon the next Chem-Math Unit #24, and neglect these mechanical aspects of physical problem set-up.

24) <u>Developing Metacognitive Strategies</u>

A) Mature, successful students constantly observe and evaluate their own thinking processes.

B) Students launch into autopilot when given word problems to solve. They try to find an equation that "fits," instead of following a logical progression of steps that leads them from current understanding to finding the appropriate relationship. They see an algorithmic approach as the most efficient route for problem-solving. They have developed this bad habit as a result of haphazard instruction in handling their mathematics in previous science courses. Typically no one has given students explicit instruction in problem-solving.

C) Coach students into a careful reading of the problem, then assembling the information in an abbreviated format before plunging into calculations. Some teachers recommend "chunking" the info by reading five words at a time and determining what those five words are saying. Drill students in these standard key steps of the problem-solving process: read carefully to understand the problem, develop a clear plan to solve the problem, don't neglect to consider recursion (iteration) as an option, execute the plan, look back to see what you did, check your work, and identify what you learned that you can use in the future.

D) References by Polya, Schoenfeld, Simon, and Hayes provide an excellent research base.

25) ***Reinforcing Study-Skills***

A) Good study-skills are essential for success in academics. They are a part of preparation for life-long learning as well as for on-the-job professional development.

B) Every teacher should take some responsibility for assisting students in acquiring good study-habits. Precocious students may never have had to use them, but their sloppy approach may fail to serve them at some juncture. Poor students may never have been properly coached. Students’ proclivity to multitask means they may not have learned to marshall and apply their resources in a focused and concerted fashion. The university campus has a multitude of support structures of which students are often unaware.

C) Present a typical junior high school lesson in study-skills; most still need it. Give them a list of resources including tutorial sessions, paid and unpaid tutors, and where and when review sessions will be held.

D) The College Board has documented many ways in which students can improve their study habits.

26) ***Addressing Motivation***

A) Without a reason to learn, to master material, and to actually gain true understanding, all instruction is for naught.

B) If a student's needs are not met, and/or if she feels that she is being dealt with unfairly, she will flip over into an irrational cope mechanism. She will seek to "get the grade" she can live with, and will focus all her academic and intellectual attention on her other classes. I call this phenomenon *Educational Darwinism.*

C) Make a personal connection with each student. Discover her life goal. Let her know that you care about her, that you have her best interests in mind, and that your goal is helping her achieve her goal. Show her how your course relates to her goal. Follow the basic rule of good teaching: find out what students already know and are able to do, and build upon that. Tell 'em what you're going to teach 'em, then teach 'em, then tell 'em what you taught them. Make sure that your summative assessments test what you taught, not something else.

D) Carol Dweck, Pintrich, and Bandura have written extensively about attitude and motivation.

27) Using Models

A) Some of the best recently-developed STEM pedagogy is model-based instruction that builds upon the learning-cycle (Lawson, Karplus, Osborne).

B) Many students see science principles as a collection of unrelated statements to memorize. They don't see them as the result of attempts by scientists and scholars to make sense of and to rationalize observations of natural and experimental phenomena.

C) Consult the POGIL literature to see how lessons are preceded by a model of which students must make sense. The forerunner to this technique is the learning-cycle (Lawson & Renner). Consult the physics education research literature to see how Hake and Halhoun used model-based pedagogy to revise the introductory physics curriculum.

D) The alphabet science courses of the post-Sputnik era, PSSC, BSSC, IPS, and Chem-Study, were all great exemplars of model-based pedagogy, especially in their laboratory programs where their models derive from experimental results. However these programs required extensive teacher-training to ensure the teachers preserved their purity as they were developed and intended. In addition PSSC and Chem-Study are not "programs for every student." (I cannot speak for BSSC.) IPS requires pure dedication to inquiry-based learning.